Dynamic Rational Inattention
and Business Cycles

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based on joint work with Filip Matějka and Mirko Wiederholt

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Background

- Macroeconomic variables typically move smoothly.

- In conventional business cycle models, the same variables tend to jump in response to shocks.

    - To capture the inertia in the data those models rely on *multiple* "frictions."

- Sims (1998) conjectured that the smoothness in the data could instead be understood as the result of a *single* friction, "rational inattention."

    - People cannot attend to all information and choose how to allocate attention.

- A DSGE model with rational inattention produces impulse responses similar to a New Keynesian model (Maćkowiak and Wiederholt, 2015).

This presentation

- A rationally inattentive agent chooses optimal signals about the state of the economy.

- In a dynamic environment the optimal signals induce a mix of backward-looking and forward-looking behavior (Maćkowiak, Matějka, and Wiederholt, 2018).

- Apply this insight in a real-business-cycle model.

  - Under rational inattention the response of the economy to a productivity shock is dampened and delayed.

  - The response to news about future productivity is simultaneously smooth and *front-loaded*.

- A parsimonious model of how business cycle disturbances propagate.

An attention problem in a dynamic world

$$\min_{K,A,B,\Sigma_\psi} E[(X_t - Y_t)^2]$$

subject to

$$X_t = \phi_1 X_{t-1} + \ldots + \phi_p X_{t-p} + \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q}$$

$$Y_t = E[X_t | \mathfrak{T}_t]$$

$$\mathfrak{T}_t = \mathfrak{T}_0 \cup \{S_1^K, \ldots, S_t^K\}$$

$$S_t^K = A X_t^M + B \varepsilon_t^N + \psi_t^K$$

$$\lim_{T \to \infty} \frac{1}{T} I(\bar{X}_0, X_1, \ldots, X_T; S_1^K, \ldots, S_T^K) \leq \kappa$$

Optimal signals

- The optimal signal is a one-dimensional signal about X_t and all other variables that help predict X_{t+1}, X_{t+2}, ...

  - AR(1) process, $X_t = \phi_1 X_{t-1} + \theta_0 \varepsilon_t$

    - $S_t = X_t + \psi_t$

  - AR(2) process, $X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \theta_0 \varepsilon_t$

    - $S_t = X_t + h X_{t-1} + \psi_t$

  - ARMA(1,1) process, $X_t = \phi_1 X_{t-1} + \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1}$

    - $S_t = X_t + h \varepsilon_t + \psi_t$

- Two incentives: learn about the current optimal action, X_t, and walk into subsequent periods well informed about X_{t+1}, X_{t+2}, ...

Implications of the optimal signals for actions

- AR(1) process, $X_t = \phi_1 X_{t-1} + \theta_0 \varepsilon_t$.

  - $S_t = X_t + \psi_t$.

  - $Y_t = E[X_t | \mathfrak{T}_t]$, satisfies:

$$E[X_t | \mathfrak{T}_t] = K\, S_t + (1 - K)\, E[X_t | \mathfrak{T}_{t-1}]$$

  and $K < 1$ implies a positive weight on the prior (**backward-looking behavior**).

- AR(2) process, $X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \theta_0 \varepsilon_t$.

  - $S_t = X_t + h X_{t-1} + \psi_t$, or

$$S_t{}' = \omega X_t + (1 - \omega)\, \underbrace{(\phi_1 X_t + \phi_2 X_{t-1})}_{best\ predictor\ of\ X_{t+1}} + \psi_t{}'$$

  and $\omega \neq 1$ implies a non-zero weight on the future (**forward-looking behavior**).

The business cycle effects of productivity shocks

- A rational inattention version of the real-business-cycle model.

- For simplicity, assume here that capital is fixed.

- Productivity follows an AR(1) process, $A_t = \rho A_{t-1} + \varepsilon_t$.

- X_t is the profit-maximizing labor input (for a firm) and the utility-maximizing labor supply (for a household).

- The wage adjusts so as to equate actual labor demand and labor supply.

- The response of hours worked to a productivity shock is dampened and delayed compared with the perfect information equilibrium.

  - Backward-looking behavior under rational inattention.

Impulse response of labor input to a productivity shock

Perfect information equilibrium
Equilibrium with rational inattention
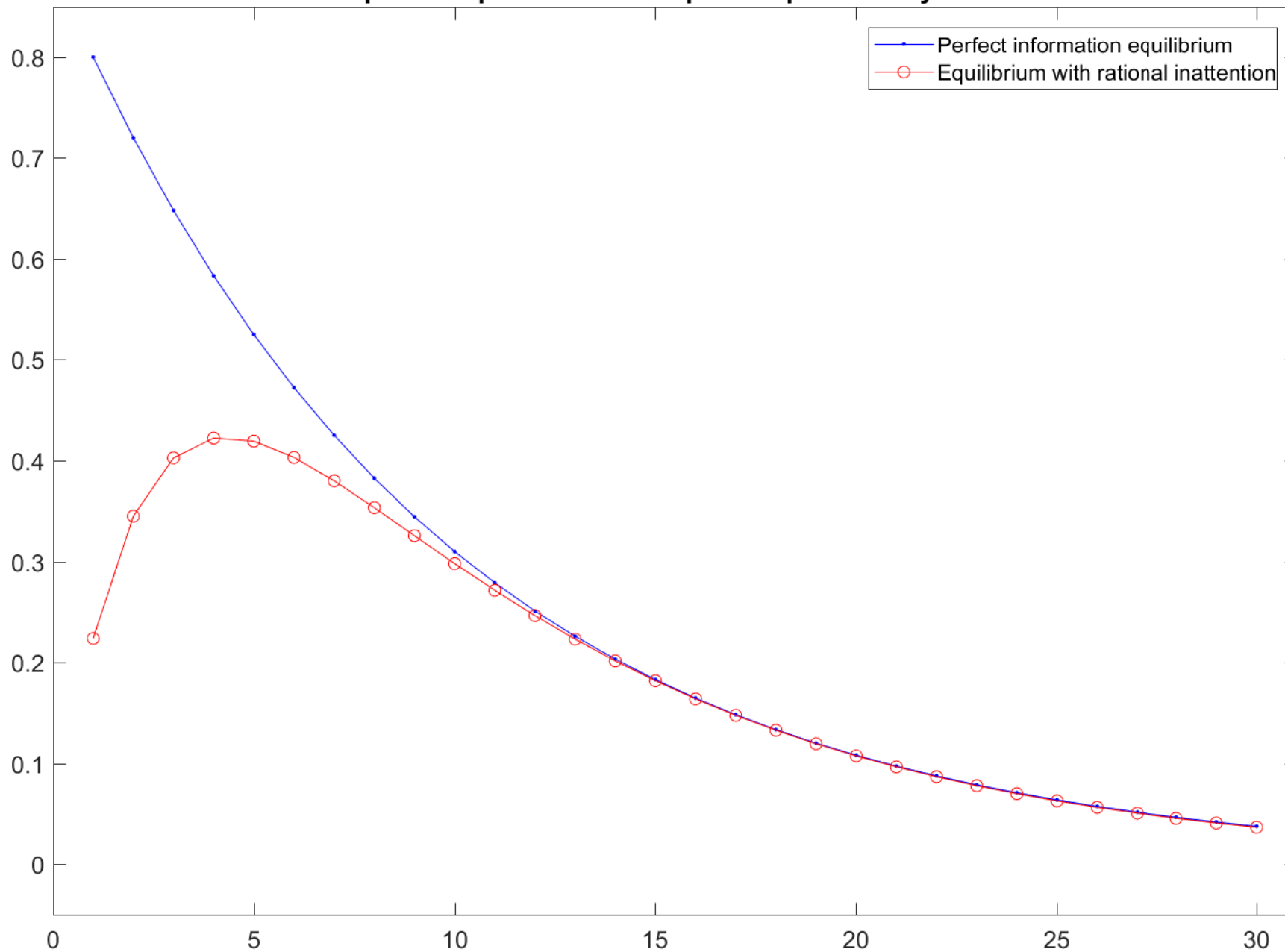
The business cycle effects of news shocks

- Productivity follows an ARMA(1,1) process, $A_t = \rho A_{t-1} + \varphi \varepsilon_{t-1}$.

- The optimal signal is

$$S_t = A_t + h\varepsilon_t + \psi_t$$

or

$$S_t{'} = \omega A_t + (1-\omega)\underbrace{(\rho A_t + \varphi\varepsilon_t)}_{best\ predictor\ of\ A_{t+1}} + \psi_t{'}$$

    – The optimal signal confounds current productivity with expected future productivity.

    – Actions respond to news about future productivity as if current productivity has changed.

- The response of hours worked to a news shock is *faster* than under perfect information.

    – Forward-looking behavior under rational inattention.

Impulse response of labor input to a news shock
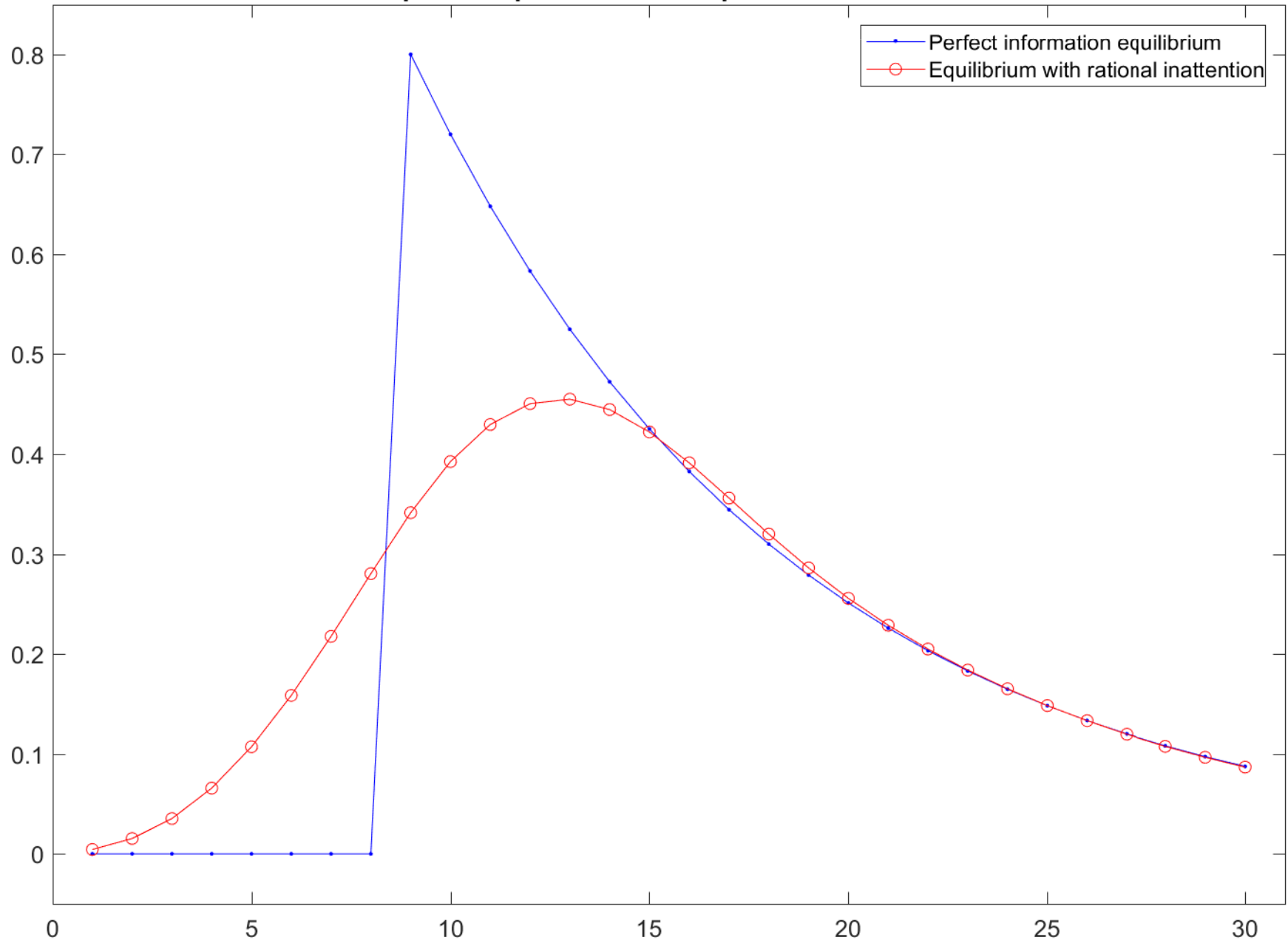
Summary

- Rational inattention induces a combination of delay in actions and forward-looking actions.

- This can help explain:

  - dampened and delayed responses to current shocks.

  - smooth and front-loaded responses to news about the future.

- A parsimonious model of how business cycle disturbances propagate.